Chapter 8. On elementary particles' spectra

1.0. Introduction

According to modern representations, all elementary particles are the bound states (including the excited states) of a small set of particles. For example, according to (Gottfried and Weisskopf, 1984): "The nucleon is simply a basic state of a compound spectrum of particles which we have named a baryon spectrum. Similarly pion is the lowest state of meson spectrum".

In present chapter we wont show how in the framework of CWED the spectra of the particles as bound and excited states of a small set of some basic particles can be formed.

1.1. The spectra of characteristics of elementary particles

Generally each elementary particle is defined by a set of various characteristics: a mass, a spin, an electric charge, the strong and weak "charges" (i.e. the characteristics, which define intensity of strong and weak interaction), the numbers of "affinity" (numbers, owing to which one family of particles differs from another - lepton, baryon and other numbers), etc.

The particles, characterized by identical characteristics, except for any one of them, create a spectrum of elementary particles regarding this variable characteristic. For example, if as such variable characteristic the mass of particles is accepted, they speak about a mass spectrum of elementary particles.

According to the modern theory there are some limiting conditions of the composition of elementary particles, which can be named the conservation laws of this characteristic: e.g. the laws of conservation of energy, momentum, angular momentum, laws of conservation of an electric charge and charges of other interactions, laws of conservation of numbers of "affinity", etc. Some laws (principles) also exist, such as a principle of uncertainty of Heisenberg, which restrict the transition from one family or a spectrum of particles to another.

As is known, the existing field theory cannot explain the appearance of elementary particle characteristics and cannot deduce the majority of conservation laws of these characteristics: they are entered as consequences of experiments.

If to speak, for example, about mass spectra of particles, the following restrictions exist:

1) according to the energy-momentum conservation law the rest free light particles cannot break up to heavier particles, but heavy particles can break up to more light particles;

2) nevertheless, according to a uncertainty principle of Heisenberg, heavy particles cannot comprise the light particles as a ready particles (for example, the neutron cannot comprise electron as a free particle).

The conclusions of the quantum theory are undoubtedly correct and was confirmed by experiments, and we should show, that they do not contradict to the results of CWED.

2.0. A hypothesis of formation of spectra of elementary particles in CWED

Within the framework of CWED the electromagnetic twirled waves (EM-particles) possess the same characteristics, as quantum elementary particles. As we saw (see above chapters), the twirled harmonic waves, appearing here, can have integer or half spin, can be charged or neutral, etc. The mass of particles within the frameworks of CWED is the "stopped" energy of the twirled standing wave. Thus, roughly speaking, to a heavy particle by our representation corresponds the twirled wave of high frequency, and to light particle - the twirled wave of lower frequency. Thus, we should explain the existence of spectra of the particles relatively to all these particularities.

To the simple harmonic waves in Classical Electrodynamics (briefly CED), the twirled harmonic waves in CWED correspond. Does exist in CED the opportunity of coexistence of several waves as some material formation - an elementary particle, in which the characteristics of various waves can be superposed?

As we know, such opportunity actually exists and it consists in the waves superposition, which leads to various forms of coexistence of normal harmonic waves and to the appearance of complex non-harmonic waves, which "consist" from harmonic waves of various frequencies.

Analogically to the representations of classical theory of EM waves, whose non-linear generalization our theory is, we assume that *the reason of complication of EM particles and of appearance of its spectra is the superposition of simple (harmonic) twirled waves, and the reason of disintegrations of particles is the disintegration of the compound twirled waves.*

The purpose of our paper will be to show that such superposition exists and its description completely corresponds to modern theoretical representations and is in full accordance with the experimental data.

Since CWED is the non-linear generalization of classical (linear) electrodynamics, it is possible to assume, that the opportunity of the mathematical description of the waves spectra creation should exist already in CED. Besides, since mathematical description of CWED completely coincides with the mathematical description of quantum electrodynamics (QED), we should show that the similar forms exist in QED as well as in CWED.

2.1. Superposition of «linear» waves

Remember that under “linear” waves we understand the waves of the linear theory.

As it is known (Grawford, 1970), the any wave can be represented by superposition of more simple waves, named “modes” (terms: “simple harmonic oscillation”, “harmonics”, “normal oscillation”, “own oscillation”, “normal mode” or simply “mode” are identical). The properties of each mode of any compound system are very similar to properties of simple harmonic oscillator.

In many physical phenomena the system motion represents a superposition of two harmonic oscillations, having various angular frequencies ω_1 and ω_2. These oscillations can, for example, correspond to two normal modes of the system, having two degrees of freedom. It is true as well for the quantum mechanical waves, described by quantum wave functions (see a known example of such system is the molecule of ammonia (Grawford, 1970)).

It is possible to illustrate this fact by the example of formation of an energy spectrum of electron in hydrogen atom. Really, the electron energy spectrum in an electron-proton system is from the general point of view a spectrum of electron masses. It is possible to speak about a basic mass (basic energy) in not excited state, and about a lot of masses of electron in the excited states, when electron receives additional portions of energy (mass). These portions are very small in comparison with the basic electron energy (mass), and we cannot consider the excited electrons as new particles. But, nevertheless, it does not exclude that these are the phenomenon of the same type as new particles’ production. The increase of electron mass occurs due to absorption of photons, and the reduction of mass takes place due to emission of photons. On the other hand, we actually cannot tell here that the electron contains a photon as a ready particle.

It is easy to show (Grawford, 1970), that the change of electron energy as a result of its excitation by a photon corresponds to a hypothesis about the appearance of new particles owing to superposition of waves.

Let's consider the steady-states of the electron in one-dimensional potential well with infinitely high walls, whose coordinates are $z=-\frac{L}{2}$ and $z=+\frac{L}{2}$. We will also assume that the electron state is defined by superposition of the basic state and the first excited state:

$$\psi(z,t)=\psi_1(z,t)+\psi_2(z,t), \tag{2.1}$$

where $\psi_1(z,t)=A_1e^{-i\omega_1 t}\cos\ k_1 z$, $k_1L=\pi$, $\psi_2(z,t)=A_2e^{-i\omega_2 t}\sin k_2 z$, $k_2L=2\pi$

The probability of electron existence in the position z in the time moment t is equal to

$$\begin{aligned}|\psi(z,t)|^2 &= \left|A_1e^{-i\omega_1 t}\cos\ k_1 z+A_2e^{-i\omega_2 t}\sin\ k_2 z\right|^2 = \\ &= A_1^2\cos^2 k_1 z+A_2^2\sin^2 k_2 z+2A_1A_2\cos\ k_1 z\cdot\sin\ k_2 z\cdot\cos(\omega_2-\omega_1)t\end{aligned}, \tag{2.2}$$

We can see that the probability expression has a term, which makes harmonic oscillations with beats frequency between two Bohr frequencies ω_1 and ω_2. The average electron position in space between the wells can be found the expression:

$$\bar{z} = \frac{\int z|\psi|^2 dz}{\int |\psi|^2 dz} = \frac{32L}{9\pi^2}\frac{A_1 A_2}{A_1^2 + A_2^2}\cos(\omega_2 - \omega_1)t \,, \qquad (2.3)$$

where the integration is from one wall up to the other .

Obviously, the frequency of radiation is defined by beats frequency. Actually, electron is charged and, consequently, it will emit out the electromagnetic radiation of the same frequency, with which it oscillates. From the equation (1) we see, that average position of a charge oscillates with beats frequency $\omega_2 - \omega_1$. Therefore the frequency of radiation is equal to beats frequency between two stationary states:

$$\omega_{rad} = \omega_2 - \omega_1 , \qquad (1.4)$$

It is easy to understand that in the framework of CWED, the non-normalized quantum wave function is simply the wave field. As a consequence of this fact, the square of this wave function (i.e. the possibility density in the framework of QED) is the energy density.

As example of such problem in framework of CWED we will consider the calculation of more general case of the interference between waves of various frequencies. We will assume, that we have two EM waves 1 and 2, having electric fields $\vec{E}_1$ and. $\vec{E}_2$. The full field in the fixed point P of space will be the superposition of $\vec{E}_1$ and $\vec{E}_2$. Using complex representation of oscillations, we will write the expression for superposition of oscillations:

$$\vec{E}(t) = E_1 e^{-i(\omega_1 t + \varphi_1)} + E_2 e^{-i(\omega_2 t + \varphi_2)} , \qquad (2.5)$$

The energy flux is proportional to average value of $\vec{E}^2(t)$ for period T of the "fast" oscillations, appearing with average frequency:

$$\begin{aligned} 2 < E^2(T) >= |E(t)|^2 &= \left| E_1 e^{-i(\omega_1 t + \varphi_1)} + E_2 e^{-i(\omega_2 t + \varphi_2)} \right|^2 = \\ &= E_1^2 + E_2^2 + 2E_1 E_2 \cdot \cos\left[(\omega_2 - \omega_1)t + (\varphi_1 - \phi_{2)}\right] \end{aligned} , \qquad (2.6)$$

As we see, the energy flux varies with relatively slow beats frequency $\omega_2 - \omega_1$.

3.0. Superposition of the twirled electromagnetic waves

Let’s try to show here that at first the superposition of the twirled electromagnetic waves exists and secondly that owing to it, it is possible to receive all those results, which are known from the theory of the linear electromagnetic waves. In other words, it is necessary to show, that in this case there are actually spectra of particles, each of which represents complication of a basic twirled wave due to it superposition with other twirled waves.

As is known, all the phenomena of superposition of waves and their disintegration are described by Fourier theory (Fourier analysis-synthesis theory), in which it is shown, that any field can be synthesized from harmonic waves or analysed to harmonic waves. We will show that Fourier theory is true in case of the twirled waves as well as in case of linear waves.

3.1. The real and complex form solutions of the wave equation, as reflection of an objective reality

As is known, the wave equation

(CED form)	(CWED form)
$\left(\frac{\partial^2}{\partial t^2} - c^2\vec{\nabla}^2\right)\vec{\Phi}(y) = 0,$	$\left[\left(\hat{\alpha}_o\hat{\varepsilon}\right)^2 - c^2\left(\hat{\vec{\alpha}}\ \hat{\vec{p}}\right)^2\right]\Phi = 0,$
where $\vec{\Phi}(y) = \{E_x, E_z, H_x, H_z\}$	where $\Phi = \begin{pmatrix} E_x \\ E_z \\ iH_x \\ iH_z \end{pmatrix}$,
	$\hat{\varepsilon} = i\hbar\frac{\partial}{\partial t}, \hat{\vec{p}} = -i\hbar\vec{\nabla}$ and $\hat{\alpha}_0$; $\hat{\vec{\alpha}}$;
	$\hat{\beta} \equiv \hat{\alpha}_4$ are Dirac's matrices

has the solution, which can be written down in the form of real periodic (in particular, trigonometric) functions, as well as in the form of complex (in particular, exponential) functions

$\vec{\Phi}(\vec{r},t) = \vec{\Phi}_0\cos(\omega\, t - \vec{k}\cdot\vec{r})$ $\vec{\Phi}(\vec{r},t) = \vec{\Phi}_0^{'}\sin(\omega\, t - \vec{k}\cdot\vec{r})$	$\Phi = \Phi_o e^{-i(\omega t \pm ky)}$ or $\begin{cases}\vec{E} = \vec{E}_o e^{-i(\omega t \pm ky)}, \\ \vec{H} = \vec{H}_o e^{-i(\omega t \pm ky)},\end{cases}$

Nowadays it is considered that the representation of the solution of the wave equation (or oscillation equation) in complex form is only a formal mathematical method, since the final solutions should be real. It was also marked, that the use of complex representation is dictated only by the reasons of convenience, since in many cases the mathematical operations with exponential functions are easier, than with trigonometric.

We have shown (see the chapter 2), that within the framework of CWED the exponential solutions have an actual meaning, if we understand them in geometrical sense as the description of motion of a wave along a curvilinear (particularly the circular) trajectory. The equivalence of both descriptions becomes clear if we remember that the circular motion can represent as sum of two linear mutual-perpendicular oscillations. (We have noted that due to this fact the solutions of the wave equations of the quantum theory are not the real but complex wave functions).

Thus, it is possible to assume, that the existence of the real and complex solutions of the wave equation indicates the existence in the nature of two types of real objects: the linear and twirled (curvilinear) waves, so that the real functions describe the linear waves, and the complex functions describe the curvilinear (twirled) waves

As is known, the functions, which describ the complex periodic and non-periodic processes of non-harmonic type can be written by the sum of harmonic functions owing to Fourier analysis-synthesis theory. It must be noted that the Fourier analysis-synthesis theory allows to work equally both with real and complex functions.

From this the extremely important conclusion follows that all tools of the Fourier analysis-synthesis theory in complex representation is the mathematical apparatus, which describ the superposition and decomposition of complex twirled waves.

In other words, the complex representation of electromagnetic waves and all mathematical apparatus of the Fourier analysis-synthesis theory represent mathematical tool of CWED in the same degree as the mathematical apparatus of the real functions of Fourier analysis-synthesis theory represents the mathematical tool of usual linear Maxwell-Lorentz theory.

*Due to above, the non-linear theory of the twirled waves is the theory in which the principle of superposition takes place as well as in the linear theory***.**

For this reason the linear Maxwell-Lorenz theory can be also written down in a complex form and it looks in such form simple and consistent. Transition from the twirled waves to linear (i.e. to one of components of the twirled wave) corresponds to transition from complex values to real .

Let us consider now some peculiarities of the Fourier analysis-synthesis theory in the case of superposition of the twirled waves.

4.0. Elementary particles as wave packets

As is known, in case of superposition of more than two classical linear harmonic waves the wave groups or wave packets are formed, which are limited in space.

In the quantum mechanics a wave packet (Physics Encyclopedia, V.1, 1960) is the concept, which denotes a matter waves' field, concentrated in the limited area. The probability to find a particle is differed from zero only in the area, occupied by a wave packet. It is possible to consider this wave field as result of superposition of the certain set of plane waves.

The possibility of composition and decomposition of plane waves is a simple result of a possibility to analyze any function in a Fourier series or Fourier integral.

It is meaningful to apply the concept of a wave packet when the wave numbers $\vec{k}$ are grouped near to some $\vec{k}_0$ with small variation $\Delta\vec{k}$, $\Delta k << k_0$, since in this case the wave packet during significant time will move as a whole, with little deformation only and with the group speed $k_0 u = \left(\frac{d\omega}{dk}\right)_{k=k_0}$, corresponding to a speed of a particle, described by this wave packet. As is known, the smearing of the wave packet does not take place if it can be decomposed on standing waves, i.e. if in the decomposition series for each vector $\vec{k}$ the vector $-\vec{k}$ with the same amplitude is also entered.

Since the superposition of linear waves leads to formation of the linear wave packets, it is logical to conclude that superposition of the twirled waves leads to formation of the twirled wave packets, i.e. to the compound electromagnetic elementary particles.

It is interesting that the representation of wave function by the Fourier series (in case of periodic function) or by the Fourier integral (in case of non-periodic function) contains the negative frequencies, which in the linear theory have no place:

Real form:	Complex form:
$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos n\omega\, t + b_n \sin(\omega\, t$	$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{-in\omega\, t}$
where a_n, b_n are the Fourier coefficients.	, where c_n are the Fourier coefficients.

As it is known (Matveev, 1985), in classical optics it take into consideration that $e^{i\omega\, t}$ describes here the complex unit vector, which rotates around the origin of coordinates in a positive direction (by a rule of the right screw). In the same time the complex unit vector $e^{-i\omega\, t}$ rotates in the negative direction. Thus, the appearance of the negative frequencies is connected with transition to the rotating complex vectors as to the basic functions of Fourier-transformation.

As a simple example of formation of a linear and non-linear wave packet, we will consider a packet formed by the equidistant rectangular frequency spectrum of waves of equal amplitudes. The description of superposition of such waves can be made in real (Grawford, 1970) as well as in a complex form (Matveev, 1985), that reflects the existence of the linear and non-linear world of particles.

We will find the exact expression for a packet $\psi(t)$ formed by superposition of *N* various harmonic components, which have equal amplitude *A*, an identical initial phase (equal to zero) and the frequencies distributed by regular intervals between the two frequencies: ω_1 and ω_2. Generally we have:

(real form)	(complex form)
$\psi(t) = A\cos\omega_1 t + {}$ $+ A\sum_{n=1}^{N-1}\cos(\omega_1 + n\delta\omega)\ t + A\cos\omega_2 t$	$\psi(t) = A\sum_{n=0}^{N-1} e^{i(\omega\ t + n\delta\omega t)}$

where $\delta\omega$ is the frequency difference of two next components, and $n = 1,2,3,...,N-1$ and $\omega_2 = \omega_1 + N\delta\omega$. This formula represents the complex wave function in the form of linear superposition of number of strictly harmonic components. It appears that this sum can be expressed in the form, which is the generalization of the case of two oscillations:

$$\psi(t) = A(t)\cos\omega_m t\,, \tag{4.1}$$

where $A(t) = A\dfrac{\sin(0{,}5N\delta\omega\cdot t}{\sin(0{,}5\delta\omega\cdot t)}$ is the variable amplitude, $\omega_m = \dfrac{1}{2}(\omega_1 + \omega_2)$ is the average frequency of a wave packet. The amplitude $A(t)$ describes a wave packet envelope. It is possible to show (Grawford, 1970) that for a wave packet, Heisenberg uncertainty principle are true, what proves their wave origin. Apparently that in the case of twirled waves this principle described the particle size limit.

Since the twirled waves already are the space limited objects, it is possible to assume, that the electromagnetic particles should be combined not from infinite Fourier series, but they should be presented by the sum of the limited number of harmonics, i.e. of the twirled waves.

To describe the synthesis of the complex particles (packets) from more simple sub-packets, we will show, that any wave packet can be presented in the form of the sum of wave sub-packets. In this case, obviously, superposition (interaction) of several big packets can be considered not as superposition (interaction) of their separate harmonic components, but as superposition of their sub-packets (particles).

Let's consider the splitting of a big packet into two sub-packets. We will present a compound wave $\psi(t)$ (see above (4.1)) in the following form:

$$\psi(t) = A\cos\omega_1 t + A\sum_{n=1}^{N-1}\cos(\omega_1 + n\delta\omega)t + A\cos\omega_2 t =$$
$$= (A\cos\omega_1 t + A\sum_{m=1}^{N_1-1}\cos(\omega_1 + m\delta\omega)t + A\cos\omega_2' t) + \quad , \tag{4.2}$$
$$+ (A\cos\omega_1' t + A\sum_{l=1}^{N_2-1}\cos(\omega_1' + l\delta\omega)t + A\cos\omega_2 t)$$

where $N = N_1 + N_2$, $\omega_2' = \omega_1 + N_1\delta\omega$, $\omega_1' = \omega_1 + (N_1 + 1)\delta\omega = \omega_2' + \delta\omega$.

Thus, we can represent the wave packet $\psi(t)$ as two sub-packets:

$$\psi(t) = \psi_1(t) + \psi_2(t), \tag{4.3}$$

where $\psi_1(t) = A\cos\omega_1 t + A\sum_{m=1}^{N_1-1}\cos(\omega_1 + m\delta\omega)t + A\cos\omega_2' t$

$$\psi_2(t) = A\cos\omega_1' t + A\sum_{l=1}^{N_2-1}\cos(\omega_1' + l\delta\omega)\ t + A\cos\omega_2 t .$$

It is convenient to enter a shortening for a packet of waves ${}^{\Sigma}\psi(t)$, where sigma means the sum of harmonic waves (in particular, a sub-packet). Then, representation of a packet in the form of the sum of sub-packets can be written down as:

$$\psi(t) = {}^{\Sigma}\psi_1(t) + ... + {}^{\Sigma}\psi_2(t) = \sum_i {}^{\Sigma}\psi_i(t), \tag{4.4}$$

From the above-stated calculations it is visible that decomposition on sub-packets (particles) is not unambiguous, since each one of the sub-packets can be grouped from harmonic waves in various ways. It is possible to assume, that the decay of the same particle on different channels can be considered as an opportunity of disintegration of packet on various sub-packets.

Using the above-stated reason it is easy to prove also that superposition (interaction) of sub-packets leads to the same consequences as interaction of separate harmonic waves, i.e. it leads to beats and to change of the energy level, independent from other non-interacting sub-packets.

Except for curvilinearity in CWED there is one more serious difference from linear electrodynamics: in CWED alongside with the full periodic twirled waves (bosons), exist also the half-period twirled waves (fermions). This creates a number of additional variants of the wave superposition, which are not present in linear electrodynamics. Besides, the curvilinearity enters into the physics one more characteristic of particles - the currents.

It is not difficult to understand also that the superposition of the twirled waves in comparison with the superposition of linear waves has more variants in a spatial arrangement of waves, and, hence, has more complex mathematical description. Actually we can see this in the case of description of hadrons (chapter 7).

It is easy to see, that the principle of superposition does not provide stability or, at least, metastability of compound electromagnetic particles. Thus, we should additionally find out the conditions of stability of the twirled waves.

5.0. The resonance theory of stability of elementary particles

As electromagnetic particles represent the spatial formations, here it is necessary to speak about spatial packets, which are formed by superposion of twirled waves of a various positions in space (e.g., by superposion of the twirled waves, which lie on three mutual-perpendicular coordinate planes).

As is known (Shpolskii, 1951), at the superposition of harmonic waves are formed the Lissajous figures of two various types. At commensurable frequencies of waves, the standing waves are formed; at incommensurable frequencies the motion of waves is refered to as quasi-periodic.

In the physics of waves and oscillations exist two sorts of the problems, leading to the appearance of the compound waves and oscillations.

An example of first type of problems is oscillation of the body volume (sphere, cylinder, torus, etc.), by which we can represent a particle. Here the suitable mechanical example is the oscillation of the sphere, prepared from a hydrophobic liquid and placed in water (for example, a sphere from mineral oil in water). In a microphysics the object, which possesses similar oscillations, is the drop model of a nucleus.

Problems concerning the same type are also the problems of oscillation of vortical rings in a perfect liquid or gas, studied by W. Kelvin (we will name conditionally such problems Kelvin's problems). In case of the oscillations of the linear vortex considered in work (Kelvin, 1867) he obtains the exact solution. Here Kelvin has compared the radiation spectra of the atoms, obtained little time before by Bunsen, to possible spectra of oscillation of vortex. Comparison of such type of oscillations with observable results is available e.g. in works (Paper collection, 1975) and (Kopiev and Chernyshev, 2000). (It is necessary to note, that in his articles W. Kelvin used the term “atoms” in sense of Democritus as the smallest indivisible constituents, i.e. in modern terminology as elementary particles).

Certain of the Kelvin significant conclusions from the paper “Atom as Vortex” we cite below:

“The author called attention to a very important property of the vortex atom. The dynamical theory of this subject require that the ultimate constitution of simple bodies should have one or more fundamental periods of vibration, as has a stringed instrument of one or more strings.

As the experiments illustrate, *the vortex atom has perfectly definite fundamental modes of vibration*, depending solely on that motion the existence of which constitutes it. The discovery of these fundamental modes forms an intensely interesting problem of pure mathematics. Even for a simple Helmholtz ring, the analytical difficulties, which it presents, are of a very formidable character. The author had attempted to work it for an infinitely long, straight, cylindrical vortex. For this case he was working out solutions corresponding to every possible description of infinitesimal vibration.

One very simple result, which he could now state is the following. Let such a vortex be given with its section differing from exact circular figure by an infinitesimal harmonic deviation of order **i**. This form will travel as waves round the axis of the cylinder in the same direction as the vortex rotation, with an angular velocity equal to (**i**-1)/**i** of the angular velocity of this rotation. Hence, as the number of crests in a whole circumference is equal to **i**, for an harmonic deviation of order **i** there are **i**-1 periods of vibration in the period of revolution of the vortex. For the case **i**=1 there is no vibration, and the solution expresses merely an infinitesimally displaced vortex with its circular form unchanged. The case **i**=2 corresponds to elliptic deformation of the circular section; and for it the period of vibration is, therefore, simply the period of revolution. These results are, of course, applicable to the Helmholtz ring when the diameter of the approximately circular section is small in comparison with the diameter of the ring, as it is in the smoke-rings exhibited to the Society.

The lowest fundamental modes of the two forms of transverse vibrations of a ring, such as the vibrations that were seen in the experiments, must be much graver than the elliptic vibration of the section. It is probable that the vibrations which constitute the incandescence of sodium-vapour are analogous to those which the smoke-rings had exhibited".

As examples of other type of problems are oscillations of sound and electromagnetic waves into various types of the closed cavities (boxes), whose surface is motionless. Such cavities refer to as closed wave-guides or resonators and consequently we will conditionally name this type of problems the closed wave-guide or resonator problems. In the classical physics a set of researches is devoted to such type of problems. Examples of such type of problems are also eigenvalues problems of wave functions in the quantum mechanics, which we will consider briefly below.

The above first and second type of problems leads to solutions of type of the standing waves, which have the relative time stability.

Thus, it is possible to assume, that stability (or the relative stability named metastability) of electromagnetic particles is connected with a formation of standing waves.

As is known, a mathematical condition of appearance of standing waves is the proportionality of wavelength to the size of box (volume), in which the wave propagates. Therefore, at the study of a possible solution of these sorts of problems the basic role the limits play, which are imposed on propagation of waves or, in other words, the boundary states, imposed on wave functions.

Below we will show that from this boundary states follow the quantization conditions of characteristics of electromagnetic elementary particles.

5.1. Photon wave equation of classical electrodynamics

Let,s consider again wave equation for the electric and magnetic field vectors (Matveev, 1989):

$$\left(\frac{\partial^2}{\partial t^2} - c^2\vec{\nabla}^2\right)F(\vec{r},t) = 0\,, \qquad (5.1)$$

where $\vec{F}$ is whichever of the EM wave functions.

The general harmonic solution of this wave equation has the complex $F(\vec{r},t) = F(\vec{r})e^{-i\omega t} = F_0 e^{i(\vec{k}\vec{r}-\omega t)}$ or trigonometric forms $F(\vec{r},t) = F_0\cos(\vec{k}\vec{r} - \omega\, t)$, where $\omega = 2\pi\nu$ is the angular frequency, ν is the linear frequency, $\vec{k} = \frac{2\pi}{\lambda}\frac{\vec{p}}{|\vec{p}|}$ is the wave vector, $k = |\vec{k}|$ called the *wave number* so that $k = \omega/\upsilon = 2\pi/\upsilon\, T = 2\pi/\lambda$, T is the wave period and λ - wavelength. Note that we will obtain the same results whether we use the real forms or the complex.

Putting this solution in (5.1) we find for $F(\vec{r})$ the following equation for stationary waves:

$$\left(\vec{\nabla}^2 + k^2\right)F(\vec{r}) = 0\,, \qquad (5.2)$$

Using these solutions it is also easy to obtain the dispersion law for EM waves:

$$\frac{\omega^2}{c^2} = k_x^2 + k_y^2 + k_z^2\,.$$

The equation (5.3) refers to as Helmholtz equation and is universal for the description of the coordinate dependence of harmonic waves' characteristics.

On the basis of this equation, was constructed the Kirchgoff diffraction and interference theory of light, which has excellently proved to be true an enormous experimental material, which can generalized in the case of the twirled waves' theory.

5.2. Wave equation solution for resonator

To analyse the electromagnetic wave equation solution for resonator we will take (Weinstein, 1957) an orthogonal box from metal with *a*, *b* and *d* sites as our model of resonator. We will show that this solution is the standing electromagnetic waves.

According to (5.2) the electric field must satisfy the equations

$$\left(\vec{\nabla}^2 + k^2\right)\vec{E}(\vec{r}) = 0$$

$$\vec{\nabla}\vec{E} = 0$$

with the boundary state $\vec{E}_{II} = 0$ at the walls of the cavity (because inside the walls the electric energy will be rapidly dissipated by currents or polarization, the electric field intensity drops rapidly to zero into the walls). However, there could be an electric field *perpendicular* to the walls, because there could be the surface charge on the wall. This gives a possible solution:

$$\begin{cases} \vec{E}_x = E_{0x}k_x \cos k_x x \ \sin k_y y \ \sin k_z z \\ \vec{E}_y = E_{0y}k_y \sin k_x x \ \cos k_y y \ \sin k_z z \, , \\ \vec{E}_z = E_{0z}k_z \sin k_x x \ \sin k_y y \ \cos k_z z \end{cases} \qquad (5.3)$$

For example, taking any x for which $\sin k_x x = 0$, the second and third terms above are identically zero, but the first term certainly isn't.

Also from $di\upsilon\vec{E} = 0$ using (5.3) we find $\vec{\nabla}\vec{E} = (E_{0x}k_x + E_{0y}k_y + E_{0z}k_z)\sin k_x x \cdot \sin k_y y \cdot \sin k_z z = 0$ if choosing $\vec{k}$ so that $\vec{k} \cdot \vec{E}_0 = 0$.

Here the wave equation requires $k_x = m\pi/a$, $k_y = n\pi/b$, $k_z = l\pi/d$, $\omega^2 = c^2\left(k_x^2 + k_y^2 + k_z^2\right)$ or $\omega = c\sqrt{k_x^2 + k_y^2 + k_z^2}$, where (l, m, n) are positive integers, e.g. (1, 1, 0) or (3, 2, 4), etc. In other words, each possible standing electromagnetic wave in the box corresponds to a point in the (k_x, k_y, k_z) space, labelled by three positive integers.

If we want also to obtain the general solution of the magnetic field, we first observe that the magnetic field satisfies the same equations and the boundary states as the electric field, and so the solution looks exactly the same as the electric solution. (An alternative way is to use $\vec{B} = \vec{\nabla} \times \vec{E}/i\omega$, which can be easily obtained from Maxwell theory).

Thus, the character of the general solution for EM wave in the cavity is the standing electromagnetic wave.

It is easy to see, that the stated above description of appearance of a resonance of the linear waves, if we make it in the complex form, will correspond to the appearance of the resonance of the curvilinear (twirled) waves.

Show now that the quantum wave equation solutions for the stationary states give the identical results.

6.0. The quantum wave equations and their solutions for stationary waves

6.1. De Broglie waves as twirled EM waves

De Broglie has assumed that material particles alongside with corpuscular properties have as well the wave properties so that to the energy and momentum of a particle in a corpuscular picture there correspond the wave frequency and wavelength in a wave picture. De Broglie has shown that in this case from relativistic transformations the parities strictly follow:

$$\varepsilon = \hbar\omega \quad \text{and} \quad \vec{p} = \frac{\hbar}{2\pi\lambda}\frac{\vec{p}}{|\vec{p}|} = \hbar\vec{k}$$

and the wave function of a material particle are described by the formulas:

$$\psi(\vec{r},t) = \psi(\vec{r})e^{-i\omega t} = \psi_0 e^{i(\vec{k}\vec{r}-\omega t)} = \psi_0 e^{\frac{i}{\hbar}(\vec{p}\vec{r}-\varepsilon t)}$$

In the case of the de Broglie wave the dispersion law it is easy to find from the following energy-momentum conservation law for a particle:

$$\frac{\varepsilon^2}{c^2} = m_0^2 c^2 + \vec{p}^{\,2}$$

Really, replacing the energy and momentum by the wave characteristics, we will receive a dispersion correlation for waves of a matter:

$$\frac{\omega^2}{c^2} = \frac{m_0 c^2}{\hbar^2} + \vec{k}^{\,2}$$

We will show below that within the framework of CWED this dispersion correlation satisfies to the equation of the twirled semi-photon, which produce the Schreodinger or Dirac equations.

6.2. Helmholtz equation for de Broglie waves

The Helmholtz equation (5.2) describes the waves of various nature in homogeneous mediums and in vacuum with constant frequency ($\omega = const$). The constancy of wavelength is not supposed here.

Planck's correlation $\varepsilon = \hbar\omega$ shows that the condition $\omega = const$ entails the equality $\varepsilon = const$. Hence, Helmholtz equation can be applied to de Broglie waves at the description of motion of corpuscles in potential fields when their full energy is constant:

$$\varepsilon = \varepsilon_k + \varepsilon_p = p^2/2m + \varepsilon_p = const\ , \tag{6.1}$$

where $\varepsilon_k = p^2/2m$ is a kinetic energy, $\varepsilon_p(\vec{r}) \equiv V(\vec{r})$ is potential energy of a corpuscle in a field. From de Broglie correlation $\vec{p} = \hbar\vec{k}$ in view of (6.1) the equality follows:

$$k^2 = \frac{2m}{\hbar^2}(\varepsilon - \varepsilon_p), \tag{6.2}$$

Substituting the expression (6.2) for k^2 in (5.3) we receive the equation:

$$\left(\vec{\nabla}^2 + \frac{2m}{\hbar^2}(\varepsilon - \varepsilon_p)\right)F(\vec{r}) = 0\ , \qquad (6.3),$$

named the Schroedinger stationary equation.

From this follows, that the existing calculation methods of the energy, momentum, angular momentum and other characteristics of particle state in the quantum field theory are calculations of resonance states of elementary particles in the various types of resonators, which in the quantum theory are usually named the potential wells. From the mathematical point of view these problems refer to as eigenvalues problems.

6.3. Quantization of state of the particle in the external field

The first calculations of quantum systems concerned the electron motion in the hydrogen atom. The formulas of quantization of electron characteristics in this case have been firstly found empirically (formulas of Balmer, Paschen, etc.) . Then, it has been shown that they turn out as consequence of conditions of Bohr quantization.

The generalization of Bohr quantization rules has been made independently by Wilson and Sommerfeld. They have shown, that in case of systems with any number of degree of freedom f it is possible to find such generalized coordinates $q_1, q_2, ..., q_f$, in which the motion of system is separated on f harmonic oscillations; in this case a known rule of oscillator quantization can be applied for any of degrees of freedom. Owing to this generalization we receive f quantum conditions:

$$\oint p_1 dq_1 = \left(n_1 + \frac{1}{2}\right)h,\ \oint p_2 dq_2 = \left(n_2 + \frac{1}{2}\right)h,\ \ldots,\ \oint p_f dq_f = \left(n_f + \frac{1}{2}\right)h \ ,(7.1)$$

where the integers $n_1, n_2, ..., n_f$ refer to as quantum numbers.

As an example of the application of these rules it can be present the results of the hydrogen-like atom calculation (Shpolskii, 1951).

As de Broglie has shown, the Bohr or Wilson-Sommerfeld rules of quantisation define the conditions of the electron wavelengths integrality on various closed trajectories. Obviously, since any field can be represented as the oscillators sum, it is necessary to consider this rule as true for any quantum systems.

It is not difficult to see that within the framework of CWED these rules are natural rules of a resonance of the twirled electromagnetic waves, if we take into account a quantization rule of their energy according to Planck-de Broglie.

The results, received according to Wilson-Sommerfeld quantization rules, have later appeared as solutions of the wave equation for standing de Broglie waves (i.e. of the Schroedinger equation) for various sorts of potential wells (Shpolskii, 1951).

Thus, Schroedinger equation is the equation for calculation of resonance states of an electron wave in potential wells (resonators) of various type, boundary of wave motion in which are defined by potential energy of the system. Note, that the boundary states are expressed here by the same way, as in the classical theory of EM field:

$$\psi(a) = 0, \ \ \psi(b) = 0, \ \ \psi(d) = 0 , \tag{7.5}$$

It is easy to show, that this problem is absolutely identical to the problem of stand EM wave in resonators (and also identical to the problem of oscillation of strings, membranes or elastic body). The distinction is that the *wave vector is not constant here, but by some complex way depends on spatial coordinates; or, in other words, the dispersion relation is here defined by the potential of system, which varies from a point to point according* (6.2).

From above follows the conditions of formation of elementary particles' spectra.

7.0. Formation of elementary particles' spectra

According to our supposition the own spectra of elementary particles in CWED must arise in the same manner as the resonance states in any wave theory. The originality in comparison with calculation of stationary states of a particle in a field of other particles (solution for Schroedinger or Dirac electron equations) consists in the fact that *in this case we have not an external field (i.e. an external potential box), but the particles' themselves are like such a box.*

It is not difficult to imagine that medium in the electromagnetic resonator can possess a dispersion, depending on spatial coordinates under the same law, as potential energy in a potential well of quantum-mechanical problem. Recollecting that within the framework of CWED the EM wave function is identical to wave function of quantum mechanics, it is easy to understand that boundary states in a quantum-mechanical problem (7.5) must coincide with the boundary states in CED and CWED.